

## Focusing on Quadratics.

### Paper folding exercise

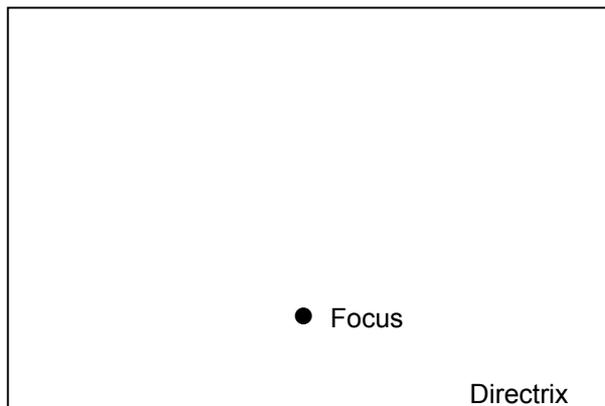
#### Step 1

Use an A4 piece of paper and place it in 'landscape' orientation. Place a clearly visible dot centered horizontally on the page and between 2 to 8 cms from the base of the page.

Label the dot: **Focus**

Label the bottom edge of the page: **Directrix**

*As shown in the diagram opposite.*

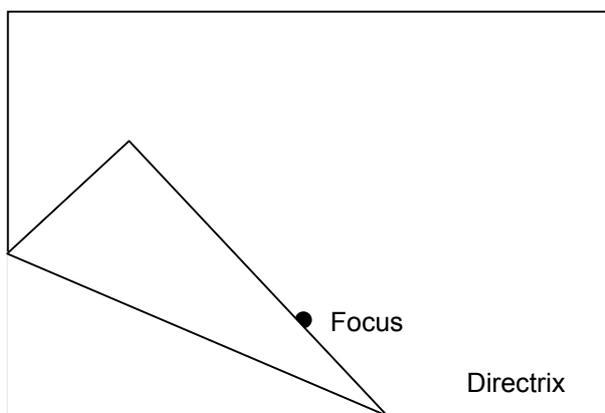


#### Step 2

Make a fold in the page such that the bottom edge of the paper (the directrix) just touches the dot (focus). Press firmly on the fold so that it leaves a permanent crease in the paper.

*As shown in the diagram opposite.*

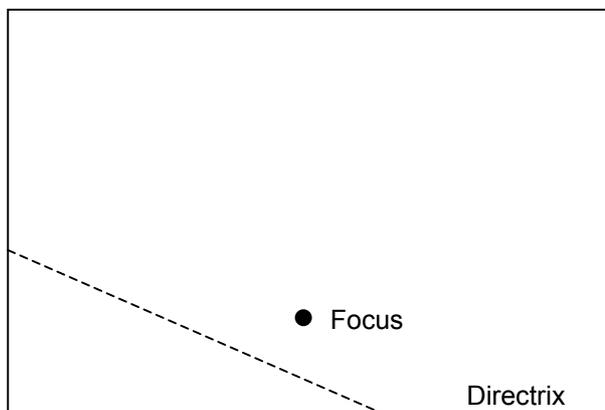
Place a small mark on the bottom of the page where the base of the page touched the focus.



#### Step 3

Unfold the paper to reveal the crease. Use a pen or pencil and a ruler to trace over the crease.

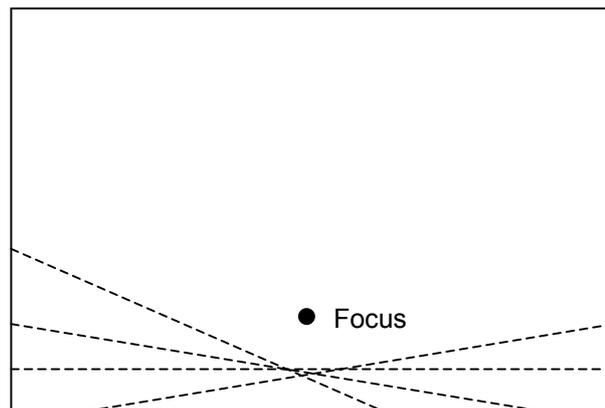
*As shown in the diagram opposite.*



#### Step 4

Repeat steps 2 and 3 until at least 30 **different** creases are formed on the page; approximately 15 on each side of the focus.

The dots covering the bottom of the page where it touched the focus should spread across the bottom edge of the page.



#### Question 1:

Describe the general shape formed by the creases in the paper.

Using a fresh piece of paper, create a single fold. When the paper is folded put a visible dot on the bottom of the page where it touches the focus.

Draw a line from the focus to this new point.

**Question 2:**

What is the angle between the line and the fold?

**Question 3:**

Measure the distance from the focus to the fold. Measure the distance from the new point to the fold. Compare the two answers.



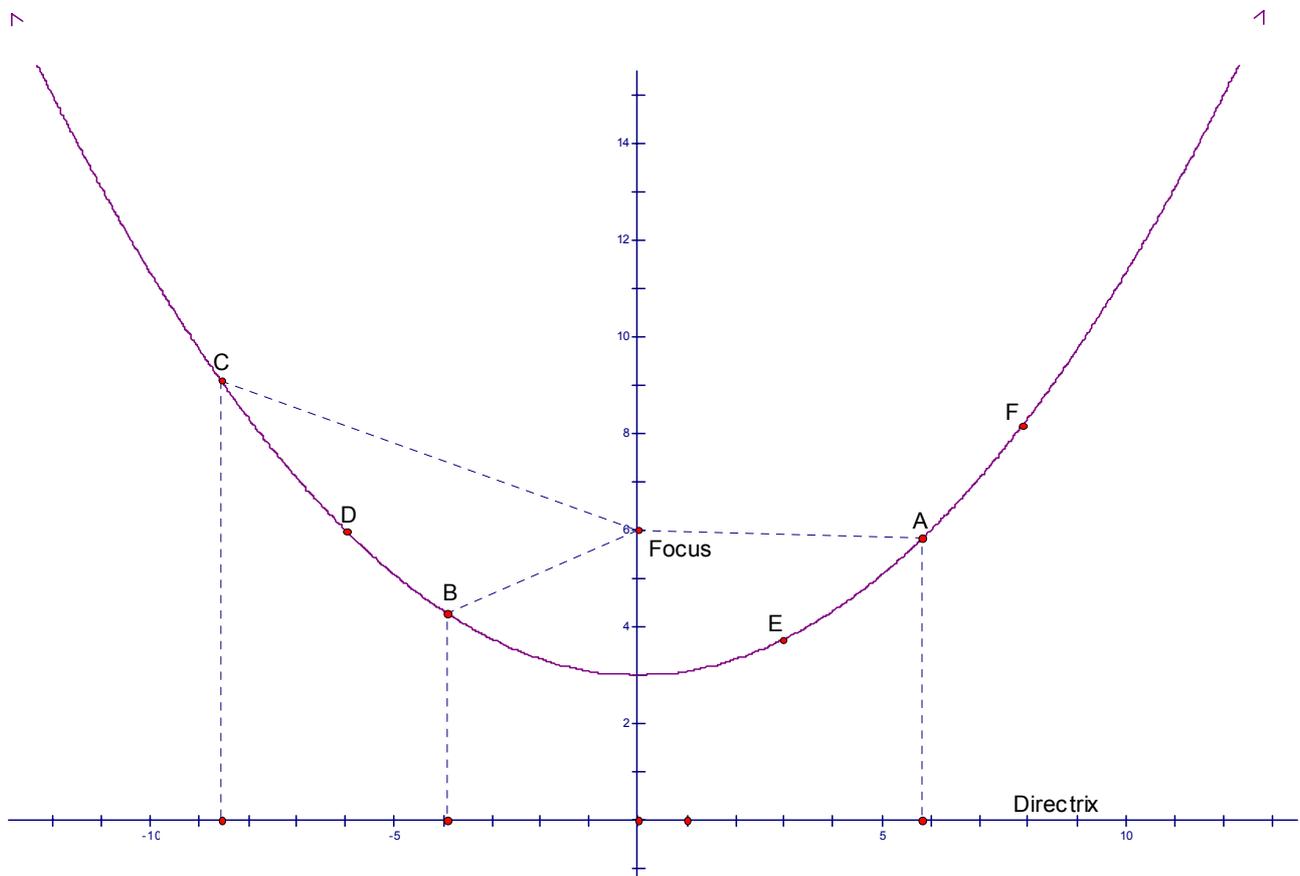
**TIP** Use the Cabri Junior App to generate the series of folds using the answers to the two previous questions as a guide to the construction. Use the 'construct locus' command to generate a series of 'folds'.

**Question 4:**

The diagram below is of a parabolic curve. Measure the distance from the focus to point A and from the directrix to point A. (Along the dotted lines). Compare the two answers.

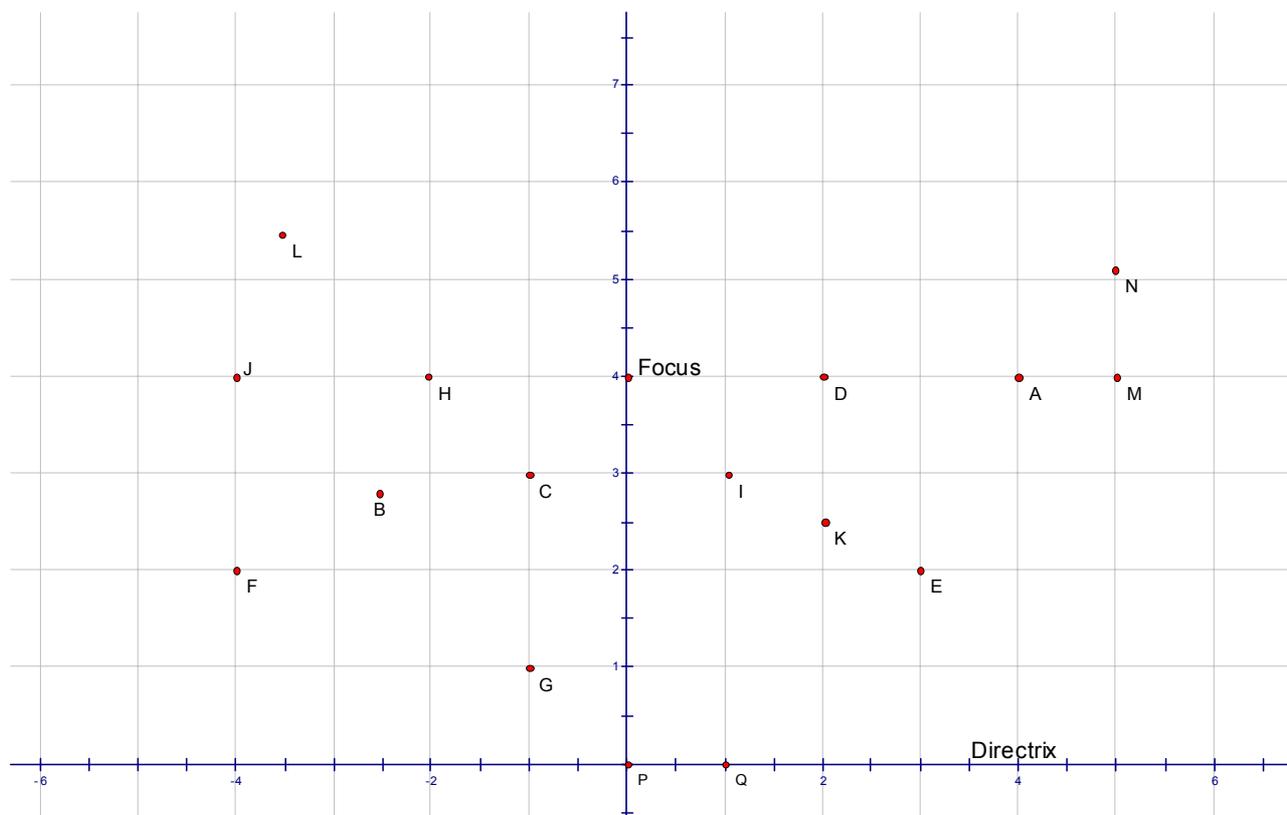
**Question 5:**

Repeat Question 4 for the points B to F.



**Question 6:**

The focus and directrix for a parabola are shown below. The parabolic curve is not shown. Use your knowledge and understanding to determine which points lay on the curve. Explain, in detail, how you determined which points are on the parabolic curve.



**TIP**

Parabolic curves can be defined algebraically. Algebraically they are known as quadratic

$$y = ax^2 + bx + c.$$

$$y = a(x - b)^2 + c$$

$$y = (x + a)(x + b)$$

Each form of the equation has its benefits, the remainder of this investigation looks at:

- How the parameters a, b and c effect each graph.
- Converting from one form of the quadratic equation to another.

**Question 7:**

Write down your own definition of a parabola.

## Further Teaching Ideas and Questions

### Question:

Use the distance properties of a parabola to determine the location a parabola with focus (0,4) and directrix  $y = 2$ . (Senior mathematics problem, excellent CAS. question).

### Demonstration:

Use a light box and parabolic reflector to show that parallel light rays parallel to the principle axis, or in a mathematical context, the  $y$  - axis, reflect and pass through the focal point.

### Question:

Use the reflective properties of a parabola to determine the location of the focus and directrix given the equation of the parabola:  $y = (x - 2)^2 + 1$ . (Senior mathematics problem, excellent CAS. question).

### Demonstration/Question:

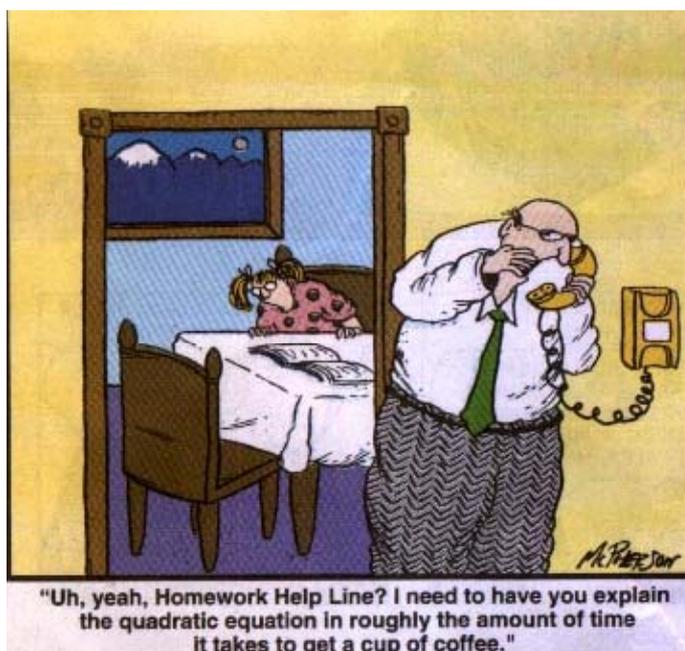
Use the graphical database on the TI-83 (Plus) to store a series of quadratic equations. Ask the students 10 quick questions "Determine the equation to each of the following parabolas." Vary the format for the equations and notice the student responses. ie: Suppose  $y_1 = (x - 3)^2 - 2$ , students will determine the equation in the same format. Similarly if  $y_2 = (x - 3)(x + 1)$ , students will generally determine the equation in its factorised form.

### Demonstration/Question:

Switch off the axis, store an equation in  $Y_0$  and ask students to determine the equation to the graph shown. The 'focus' of the problem becomes the translation of the student's guess to the hidden equation. Notice the teacher's equation is in  $Y_0$  so that it is not revealed when equation editor is displayed.

### Demonstration/Question:

A variety of techniques can be used to demonstrate the translation of functions. TI-InterActive! , Excel, Geometer's Sketchpad, Transformations App (TI-83Plus) or the Internet all represent interactive means for investigating the transformations of parabolas.

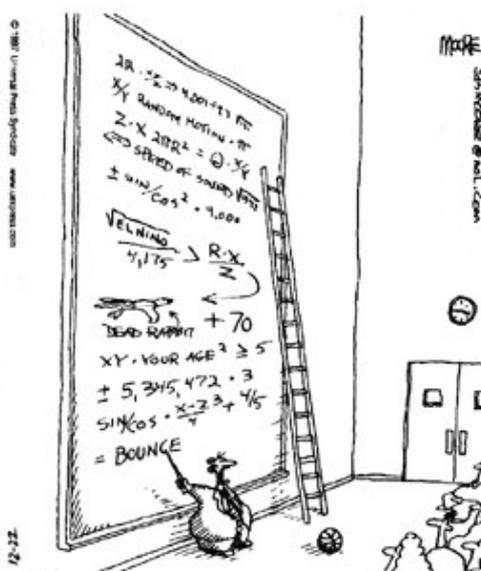


## Ball Bounce Investigation.

Getting Started – Collecting the data:

- Use the CBL/CBR Application on the TI-83Plus or load the CBR *Ranger* program onto the TI-83.
- Make sure the ball is well inflated.
- Start the *Ranger* program.
- From the main menu select: APPLICATIONS.
- Select 'metres' for the unit of measurement
- Select 'BALL BOUNCE' from the applications menu.

IN THE BLEACHERS By Steve Moore

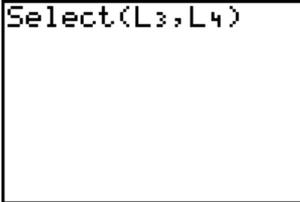
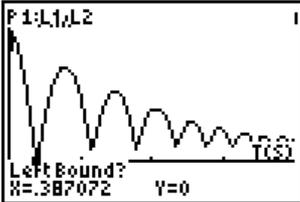
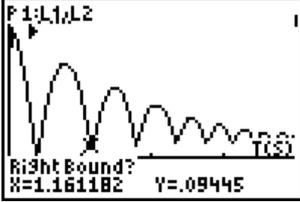
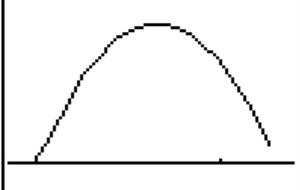
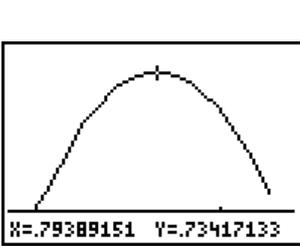
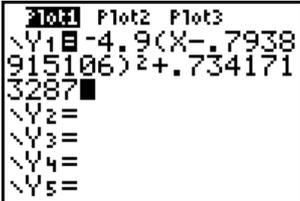
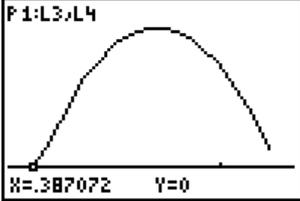


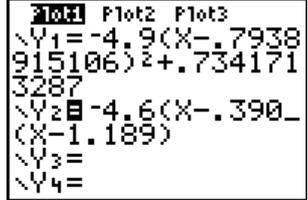
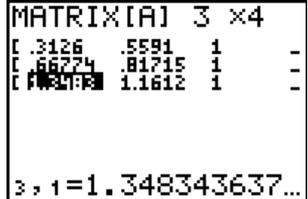
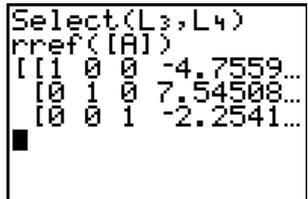
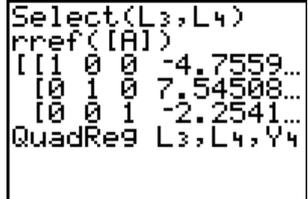
"And that, ladies and gentlemen, is the way the ball bounces."



- Follow the instructions on the calculator screen. The CBR may be detached during the experiment and returned later for data transfer.
- Hold the CBR approximately 0.5 meters above the ball. Try to hold the CBR at a constant height.
- Press the **trigger** button on the CBR. The CBR will start ticking; release the ball beneath the CBR.
- When the CBR has stopped ticking the data collection is complete.
- Return the CBR to the calculator, plug it in and press **ENTER**; a graph of the data will be displayed. Press **ENTER** when you have finished viewing the graph.
- If the data/graph is not as clear as you would like, select option 5: REPEAT SAMPLE

**Working with the data:**

Instruction	Screen Shot
<p>It is easy to remove unwanted data using the 'select' command in the List - Operations menu on the TI-83 (Plus)</p> <p><math>\boxed{2nd} \boxed{STAT} \boxed{\blacktriangleright} \boxed{8}</math></p> <p>In this example the data is stored in L<sub>1</sub> and L<sub>2</sub> on the calculator. A copy of the required data will be stored in L<sub>3</sub> and L<sub>4</sub>.</p> <p><math>\boxed{2nd} \boxed{3} \boxed{,} \boxed{2nd} \boxed{4} \boxed{)}</math></p> <p>Press <math>\boxed{ENTER}</math></p>	
<p>Use <math>\boxed{\blacktriangleright}</math> to move to the start of one of the ball bounces.</p> <p>Press <math>\boxed{ENTER}</math> to select the starting point of the bounce.</p>	
<p>Continue to use <math>\boxed{\blacktriangleright}</math> to move to the end of the selected ball bounce.</p> <p>Press <math>\boxed{ENTER}</math></p> <p>Automatically change Plot 1 to use L<sub>3</sub> and L<sub>4</sub></p>	
<p>Notice the calculator has automatically changed Plot 1 to use L<sub>3</sub> and L<sub>4</sub>. To zoom in on the data press <math>\boxed{ZOOM} \boxed{9}</math> (ZoomStat)</p> <p>This same process can be used to remove unwanted data from a large data set.</p>	
<p>Use the <math>\boxed{TRACE}</math> command to locate the top of the ball bounce.</p> <p><b>Note:</b> You may simply use the <math>\boxed{\blacktriangleup} \boxed{\blacktriangledown} \boxed{\blacktriangleleft} \boxed{\blacktriangleright}</math> arrow keys to locate the top of the ball bounce if a single data point is not close enough.</p> <p>Write down the co-ordinates of the data point or use the RCL command on the calculator to enter the equation in Y<sub>1</sub> in the form: <math>y = a.(x - b)^2 + c</math>.</p> <p>You will need to experiment with the value of <b>a</b>.</p> <p><b>Teacher hint:</b> You may also consider that the acceleration for the ball is approximately -9.8m/s<sup>2</sup> and the displacement time can be found using integral calculus.</p>	
<p>Enter the equation in Y<sub>1</sub> and then press <math>\boxed{GRAPH}</math></p>	
<p>Use the same approach above to determine the approximate co-ordinates of the x intercepts for the ball bounce. It is very rare that a data point is actually collected such that y = 0, however, estimation provides excellent results.</p>	

<p>Switch equation <math>Y_1</math> off so that you can see the graph of <math>Y_2</math> clearly.</p>	
<p>Simultaneous equations can be used to determine the equation in the form <math>y = ax^2 + bx + c</math>                  The three points selected in this example were: (0.559, 0.478); (0.817, 0.736) and (1.161, 0.095). These points have been entered into an equation in Echelon form.</p>	
<p>Using RREF (Row Reduction in Echelon Form) the equation for this parabola is given by: <math>y = -4.76x^2 + 7.545x - 2.254</math>                  Enter your equation in <math>Y_3</math></p>	
<p>Finally, use quadratic regression to determine the equation in the form: <math>y = ax^2 + bx + c</math>                  [STAT] [right] [5] [2nd] [3] [left] [2nd] [4] [.] [VARS] [right] [1] [4] then press [ENTER], the equation is determined and copied into <math>Y_4</math>.</p>	

The 'best' equation for the ball bounce data may be the one that passes the closest to all the data points.

Write down the co-ordinates of at least four points that represent your first ball bounce.

Complete the table below for each of your equations and each of your data points. Substitute the x – ordinate from each of your coordinates and compare the predicted position (y – value) with the actual coordinate.

Note: You may use  $Y_1(x)$  to determine the corresponding y - ordinate.

Equation:	Point 1:	Point 2:	Point 3:	Point 4:
$y = a(x - b)^2 + c$				
$y = a(x - b)(x - c)$				
$y = ax^2 + bx + c$ (Simultaneous)				
$y = ax^2 + bx + c$ (Regression)				

